CORRIGE DU SUJET 2

CORRIGE DE L'EXERCICE 1

géométrie du demi-cercle

$$S_1 = \frac{\pi r^2}{2} \; Y_1 = \frac{4r}{3\pi} \; Z_1 = r \; I_{yz1} = 0$$

géométrie du triangle

$$S_2 = rb \; Y_2 = \frac{b}{3} \; Z_2 = \frac{2r}{3} \; I_{yz2} = -\frac{b^2(2r)^2}{72}$$

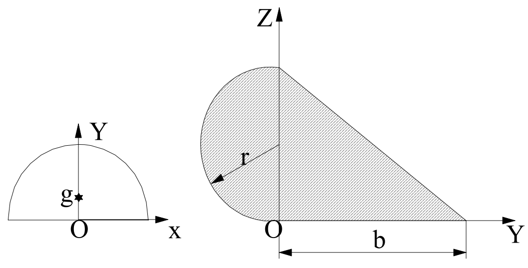
Le moment d'inertie centrifuge de la section entière

$$I_{yz} = \sum I_{yzi} + \sum S_i Y_i Z_i$$

$$I_{yz} = 0 \Rightarrow 0 - \frac{b^2(2r)^2}{72} + \frac{\pi r^2}{2} \times \left(-\frac{4r}{3\pi}\right) \times r + rb \times \frac{b}{2} \times \frac{25}{3} = 0$$

$$\Rightarrow \frac{b^2(2r)^2}{18} - \frac{2r^4}{3} + \frac{2b^2r^2}{9} = 0$$

$$\Rightarrow \frac{b^2}{6} - \frac{2r^2}{3} = 0 \Rightarrow b = 2r$$



(a) (b)

Figure 1

Moments d'inertie principaux

Pour b = 100 mm et r = 80 mm

S_1 = 10053 mm², Y_1 = -33.95 mm, Z_1 = 80 mm

$I_{zz1} = 4.5 \times 10^6$ mm^4, $I_{yy1} = 16.08 \times 10^6$ mm^4

S_2 = 2000 mm², Y_2 = -33.33 mm, Z_2 = 53.33 mm

$I_{yz2} = 4.44 \times 10^6$ mm^4, $I_{yy1} = 11.28 \times 10^6$ mm^4

Les coordonnées du centre de gravité

$$Y_G = \frac{\sum S_i Y_i}{\sum S_i} = -4.13\text{mm} \qquad Z_G = \frac{\sum S_i Z_i}{\sum S_i} = 68.18\text{mm}$$

$$I_y = \sum I_{yi} + \sum S_i (Z_G - Z_i)^2$$

$$\Rightarrow I_y = (16.08 + 11.38)\times 10^6 + 10053\times(-11.82)^2 + 2000\times(14.85)^2 = 29.3\times 10^6 \text{mm}^4$$

$$I_z = \sum I_{zi} + \sum S_i (Y_G - Y_i)^2$$

$$\Rightarrow I_z = (4.5 + 4.44)\times 10^6 + 10053\times(29.82)^2 + 2000\times(-37.42)^2 = 20.68\times 10^6 \text{mm}^4$$

$$I_{yz} = \sum I_{yzi} + \sum S_i (Y_G - Y_i)(Z_G - Z_i)$$

$$\Rightarrow I_{yz} = -\frac{100^2 \times 160^2}{72} + 10053\times(29.82)(-11.82) + 2000\times(-37.42)(14.85) = -8.21\times 10^6 \text{mm}^4$$

Moments d'inertie principaux et leur orientation

$$\text{tg}2\alpha_0 = \frac{2\times(-8.21)}{20.64 - 29.26} \Rightarrow \alpha_0 = 31.15°$$

$$I_{1,2} = \frac{29.26 + 20.64}{2} \pm \sqrt{\left(\frac{29.26 - 20.64}{2}\right)^2 + (-8.21)^2} \Rightarrow \begin{array}{l} I_1 = 34.22\times 10^6 \text{mm}^4 \\ I_2 = 15.68\times 10^6 \text{mm}^4 \end{array}$$

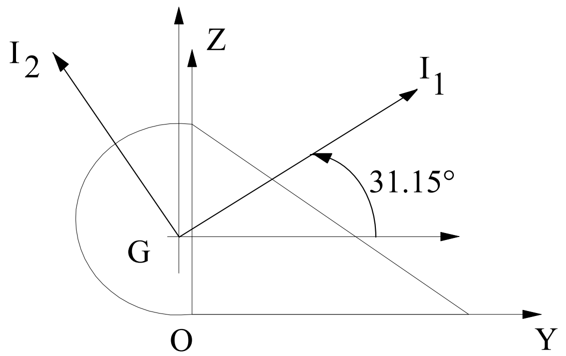
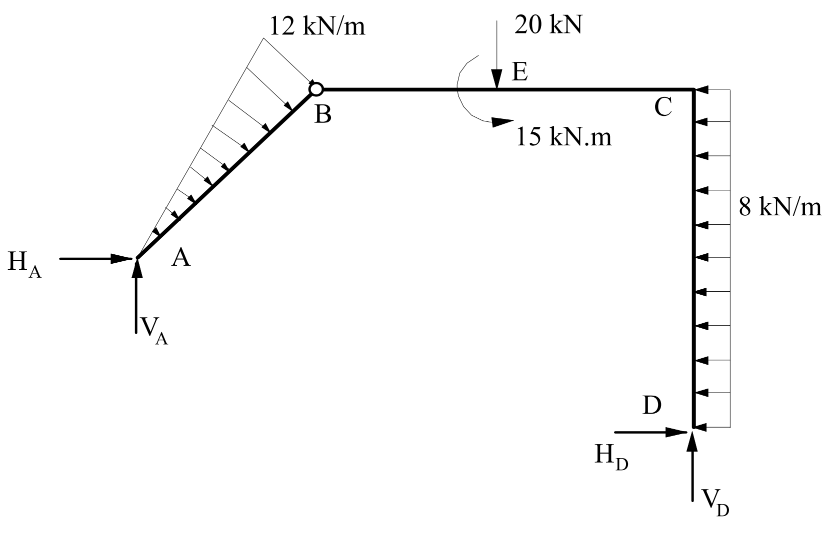


Figure 2

CORRIGE DE L'EXERCICE 2



Détermination des réactions :

$$\sum M_{/\,Bgauche} = 0 \Rightarrow 12\sqrt{2} \times \frac{2\sqrt{2}}{3} + 2H_A - 2V_A = 0$$

$$\Rightarrow 8 + H_A - V_A = 0$$

$$\sum M_{/D} = 0 \Rightarrow 6V_A + 2H_A - 12\sqrt{2} \times \frac{2\sqrt{2}}{3} - 2 \times 20 - 15 - 2 \times 8 \times 4 = 0$$

$$\Rightarrow 6V_A + 2H_A = 135$$

De (1) et (2) $\Rightarrow V_A = 18.88$ kN et $H_A = 10.88$ kN

$$\sum F \uparrow = 0 \Rightarrow V_A + V_D = 20 + 12\sqrt{2}\,\frac{\sqrt{2}}{2}$$

$$\Rightarrow V_D = 13.12\text{kN}$$

$$\sum F \rightarrow = 0 \Rightarrow H_A + H_D = -12\sqrt{2}\,\frac{\sqrt{2}}{2} + 8 \times 4$$

$$\Rightarrow H_D = 9.12\text{kN}$$

Diagrammes des efforts internes:

Tronçon AB: $0 \le x \le 2\sqrt{2}$

$$N + 10.88 \times \frac{\sqrt{2}}{2} + 18.88 \times \frac{\sqrt{2}}{2} = 0 \Rightarrow N = -21\text{kN}$$

$$T + 10.88 \times \frac{\sqrt{2}}{2} - 18.88 \times \frac{\sqrt{2}}{2} + \frac{12x}{2\sqrt{2}} \times \frac{x}{2} = 0 \Rightarrow T = 5.66 - 2.12x^2\text{kN}$$

$$M + 10.88\frac{\sqrt{2}}{2}x - 18.88\frac{\sqrt{2}}{2}x + \frac{2.12}{3}x^3 = 0 \Rightarrow M = 5.65x - 0.71x^3$$

$T(x) = 0 \Rightarrow x = 1.63$ m $\Rightarrow$ Mmax = M (1.63) = 6.13 kN.m

Tronçon BE: $0 \le x \le 2$

$$N + 10.88 + 12\sqrt{2} \times \frac{\sqrt{2}}{2} = 0 \Rightarrow N = -22.88\text{kN}$$

$$T - 18.88 + \frac{\sqrt{2}}{2} \times 12\sqrt{2} = 0 \Rightarrow T = 6.88\text{kN}$$

$$M - 18.88\frac{\sqrt{2}}{2}(2 + x) + 10.88 \times 2 + 12\sqrt{2}\left(\frac{2\sqrt{2}}{3} + \frac{\sqrt{2}}{2}x\right) = 0 \Rightarrow M = 6.88x$$

Tronçon CE: $0 \le x \le 2$

$$N - 9.12 + 8 \times 4 = 0 \Rightarrow N = -22.88kN$$

$$T + 13.12 = 0 \Rightarrow T = -13.12kN$$

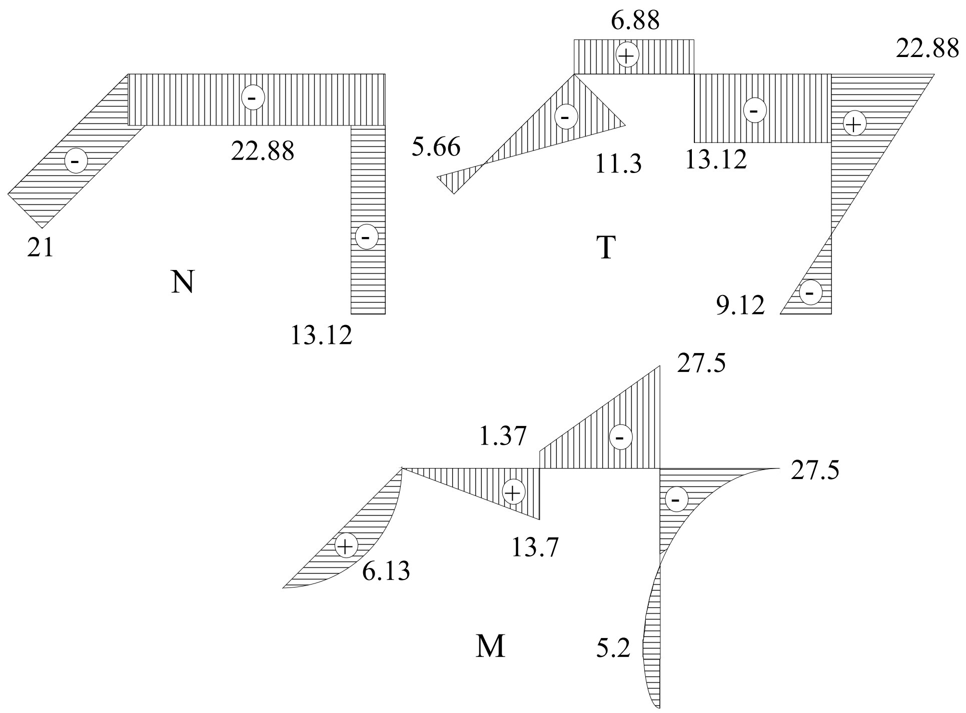
$$M - 13.12x - 4 \times 9.12 + 8 \times 4 \times 2 = 0 \Rightarrow M = 13.12x - 27.50$$

Tronçon DC: $0 \le x \le 4$

$$N + 13.12 = 0 \Rightarrow N = -13.12kN$$

$$T + 9.12 - 8x = 0 \Rightarrow T = 8x - 9.12kN$$

$$M - 9.12x + 8 \times \frac{x^2}{2} = 0 \Rightarrow M = 9.12x - 4x^2$$



CORRIGE DE L'EXERCICE 1

a) Détermination des contraintes:

$$\sigma_x = \frac{E(\varepsilon_x + \nu\varepsilon_y)}{1-\nu^2} = \frac{2.07 \times 10^5 (220 + 0.3 \times 160) 10^{-6}}{1 - .09} = 60.96N/mm^2$$

$$\sigma_y = \frac{E(\varepsilon_y + \nu\varepsilon_x)}{1-\nu^2} = \frac{2.07 \times 10^5 (160 + 0.3 \times 220) 10^{-6}}{1 - .09} = 51.41N/mm^2$$

b) Calcul de la déformation sur la jauge B:

$$\varepsilon_B = \frac{\varepsilon_x + \varepsilon_y}{2} = \frac{220 + 160}{2} \times 10^{-6} = 190 \times 10^{-6}$$

c) Détermination de la contrainte tangentielle maximale:

Les contraintes normales étant uniformes, donc elle sont des contraintes principales et $\tau_{xy} = 0$ dans ce cas. La contrainte tangentielle maximale s'écrit alors:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{60.96 - 51.41}{2} = 4.78 N/mm^2$$

d) Détermination de la variation du volume:

$$\frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

avec

$$\varepsilon_z = \frac{\nu(\sigma_x + \sigma_y)}{E} = \frac{0.3(60.96 + 51.41)}{2.07 \times 10^5} = 162.9 \times 10^{-6}$$

d'où

$$\Delta V = (\varepsilon_x + \varepsilon_y + \varepsilon_z)V = (220 + 160 + 162.9) \times 10^{-6} \times 1 \times 1.5 \times 0.2 = 1.63 \times 10^{-4} m^3$$